

## Beam combining via Orientational Stimulated Scattering: Numerical modeling and analytic solutions

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Many present-day solid-state lasers can generate very large CW power, especially in the regime of Master Oscillator – Power Amplifier (MO-PA) scheme. The task of combining individual beamlets into one high-power beam of diffraction quality is therefore quite important. One of the ways of beam combining and clean-up has been suggested recently [1]. It is based on the use of Stimulated Orientational Scattering in a Nematic Liquid Crystal (NLC) [2, 3]. The present work is devoted to the development of numeric and analytic tools of study of such beam combining.

One-dimensional (+ time) model of orientational scattering is based on the system of coupled wave equations for amplitudes  $A$  and  $B$  of waves of two polarizations:

$$\partial A / \partial z = i h B(z, t) \cdot \theta^*(z, t), \quad \partial B / \partial z = i h A(z, t) \cdot \theta(z, t), \quad \partial \theta / \partial t + \Gamma \theta(z, t) = A^*(z, t) B(z, t).$$

Here  $1/\Gamma$  is relaxation time of the grating  $\theta(z, t)$  of orientation in NLC, and the constant  $h$  determines the strengths  $\mu$  and  $\nu$  (1/meter) of cross-phase modulation for the pair of waves  $A_0$  and  $B_0$  of the same frequency:  $\partial \phi_A / \partial z = (h/\Gamma) |B_0|^2 = \nu$ ,  $\partial \phi_B / \partial z = (h/\Gamma) |A_0|^2 = \mu$ . The same quantities  $\mu$  and  $\nu$  determine gain coefficients (1/meter, with respect to intensity) of optimally frequency-shifted small signals in the presence of the opposite-polarized pumps, and the optimum frequency shift is  $\Omega_{\text{opt}} = \Gamma$ . The above coupled equations are easily generalized to account transverse ( $x, y$ ) structure of the field and diffraction effects.

We solved the system of  $z$ -equations with the use of Runge-Kutta 4-th order scheme, and the equation for  $\partial \theta / \partial t$  via first order Euler scheme. The values of the grating  $\theta$  at the mid-points with respect to  $z$ -mesh, which are required by the Runge-Kutta scheme, were taken as arithmetic averages of  $\theta$ -values at the integer points of the mesh.

We verified our solutions by comparison with the results of analytic consideration of perturbation theory, where unperturbed solution described two waves of identical frequencies producing cross-phase modulation:

$$A = A_0 \exp(i\nu z) \cdot [1 + \alpha(z, t)], \quad B = B_0 \exp(i\mu z) \cdot [1 + \beta(z, t)], \quad \theta(z, t) = (A_0^* B_0 / \Gamma) \cdot \exp[i(\mu - \nu)z] \cdot [1 + \psi(z, t)].$$

Linearized system of six equations: four  $\partial/\partial z$ -equations for  $\alpha, \alpha^*, \beta$  and  $\beta^*$  and two  $\partial/\partial t$ -equations for  $\psi$  and  $\psi^*$ , was solved for the common  $z$ - and  $t$ -dependence of the form  $\exp(i\Omega t + i\Lambda z)$  for all six functions. Remarkably, all four eigenvalues of  $\Lambda$  could be explicitly found. With the notation  $D = 1/(1 + i\Omega/\Gamma)$ , eigenvalues are:

$$\Lambda_{1,2} = 0, \quad \Lambda_{3,4} = \pm \{ (1-D) \cdot [(\mu^2 + \nu^2)(1-D) + 2\mu\nu(1+D)] \}^{1/2}.$$

When only one of the waves has large intensity, e.g.  $\mu \gg \nu$ , imaginary part of  $\Lambda_3$  describes the process of amplification of a weak signal  $\beta$  in the presence of strong pump  $|A_0|^2$ . However, for comparable intensities of two background waves the results are modified considerably.

In the talk we will present the results of numeric modeling of beam combining and cleanup.

1. I. V. Ciapurin, L. B. Glebov, C. Tsai, M. C. Stickley, B. Ya. Zeldovich, CLEO-2003 talk # CWJ4
2. B. Ya. Zeldovich, A. V. Sukhov, N. V. Tabirian, The Orientational Optical Nonlinearity of Liquid Crystals, Special issue of Mol. Cryst. & Liq. Cryst. 136, #1, 140 pages, 1986.
3. I. C. Khoo, Liquid crystals: physical properties and nonlinear optical phenomena New York, N.Y., Wiley & Sons, 1995.

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# Beam Combining via Orientational Stimulated Scattering: numerical modeling and analytical solutions

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# Overview

**Introduction:      Goals of the beam combining**

- **General idea of Orientational Nonlinearity in Liquid Crystals (ON-LC)**
- **Grating-type Nonlinearity in LC (GRON-LC)**
- **Energy transfer via GRON-type Stimulated Scattering**
- **Stability in the absence of thermal fluctuations in LC: analytic solution.**
- **Stability in the presence of thermal fluctuations: numerical modeling**

**Conclusion:      Good perspectives of beam combining**

# **High-power CW solid-state and fiber lasers are here !**

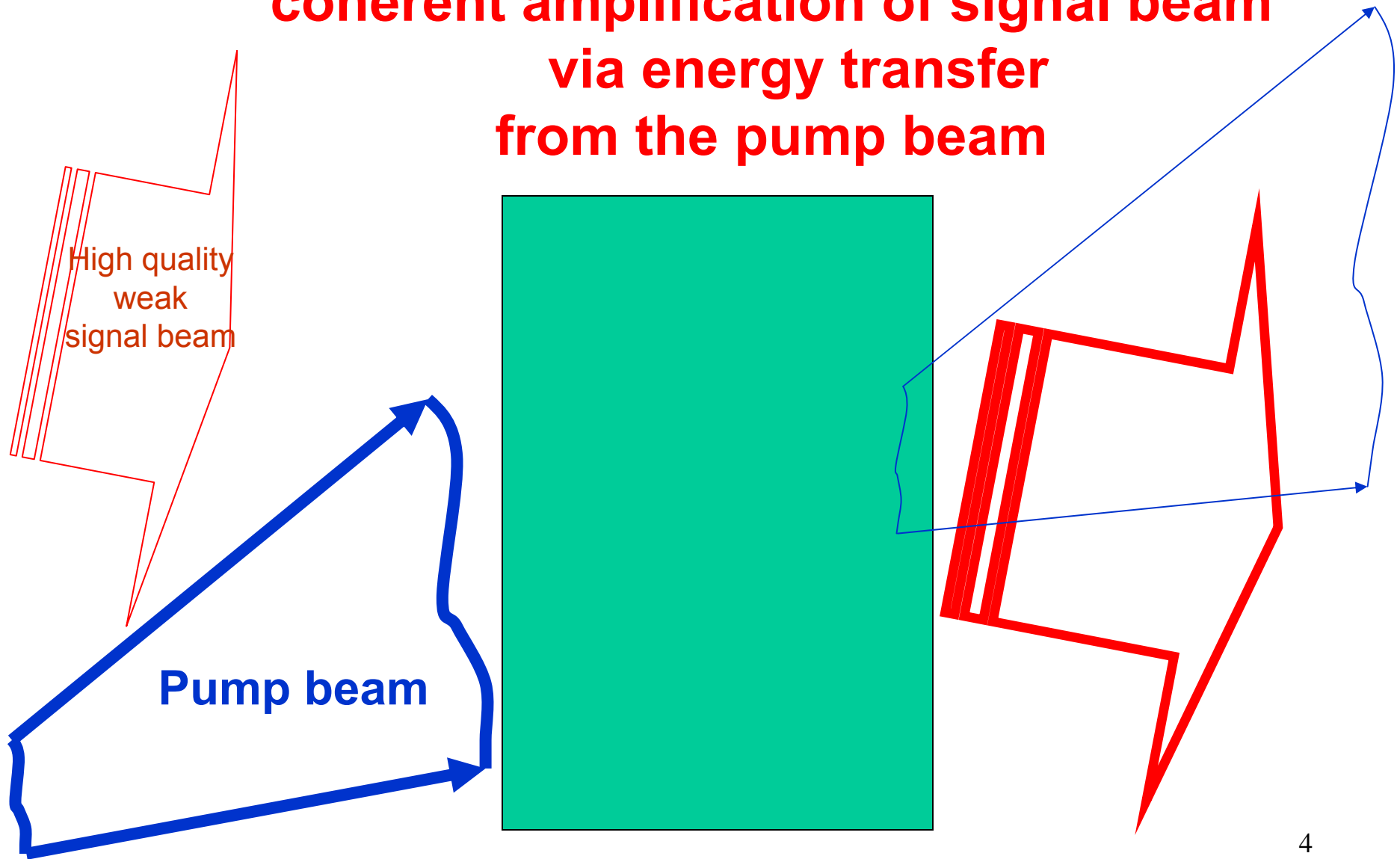
**They have the power more than 1000 Watt now  
and promise much more.**

**Beam clean-up and combining the energy from  
several amplifiers look attractive.**

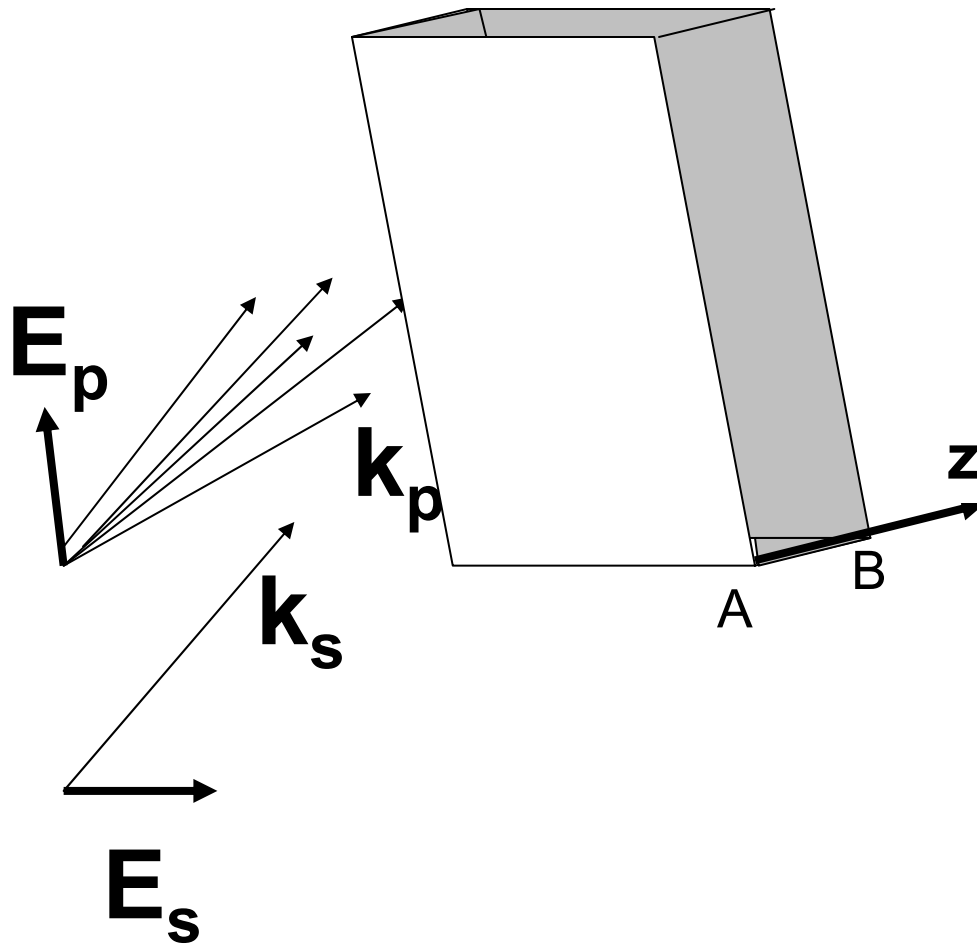
**However, Louiville's (Lagrange-Helmholtz) theorem  
does not permit to increase brightness  
by linear-optical devices.**

# Stimulated Scattering:

**coherent amplification of signal beam  
via energy transfer  
from the pump beam**



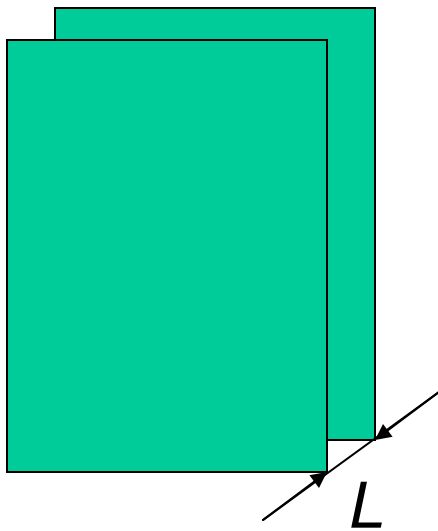
**Studied in this work:**  
**the use orientational Stimulated**  
**Scattering in a Nematic Liquid Crystal**



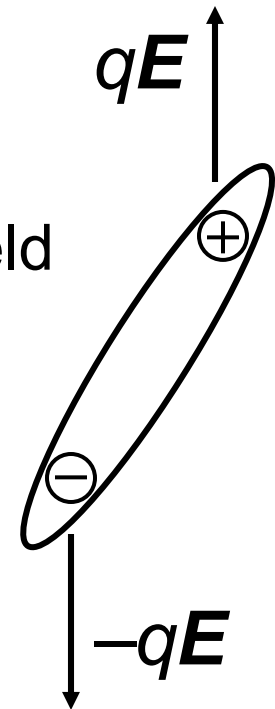
Signal plane wave  $E_s$  and inhomogeneous pump wave  $E_p$  have slightly different frequencies,  $\omega_p - \omega_s = \Omega$ . They illuminate liquid crystal cell with a planar orientation of the director.

# General idea of Orientational Nonlinearity in LC

Nematic Liquid Crystal (NLC),  
a liquid where anisotropic molecules support  
each other's orientation in a common direction  $\mathbf{n}$ ,  
“director” of NLC.



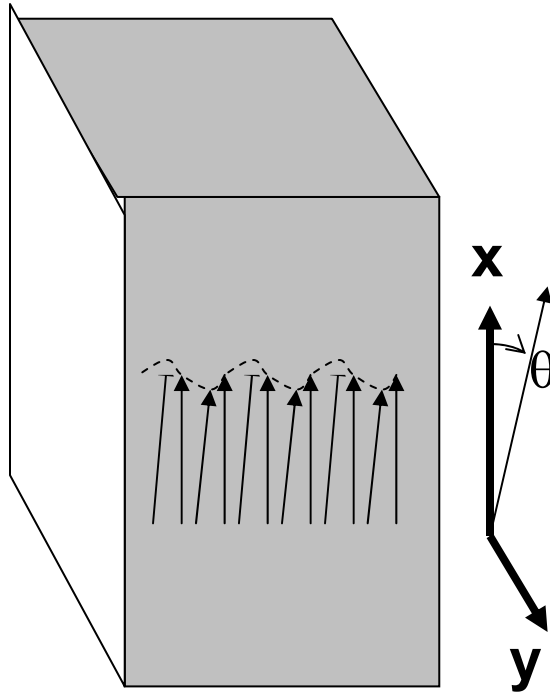
Torque due to the e.-m. field  
changes the orientation  
of the director.



Depending on the spatial size of  
director's distortions,  $100\ \mu\text{m}$  to  $1\ \mu\text{m}$ ,  
the Poynting vector of incident light  
from  $100\ \text{Watt}/\text{cm}^2$  to  $1\ \text{MWatt}/\text{cm}^2$  is sufficient  
to yield considerable nonlinear optical effects.



# Grating-type Orientational Nonlinearity (GRON) in a Nematic Liquid Crystal



Interference of  $\mathbf{E}_s \equiv \mathbf{B}$  and  $\mathbf{E}_p \equiv \mathbf{A}$  yields the grating with director orientation  $\delta \mathbf{d} \propto \exp[i\Omega t - i(\mathbf{k}_p - \mathbf{k}_s) \cdot \mathbf{r}]$ . Scattering of the pump  $\mathbf{E}_p \equiv \mathbf{A}$  by this grating of dielectric permittivity results in amplification of the signal  $\mathbf{E}_s \equiv \mathbf{B}$ .

References on stimulated scattering in LC:

- N. V. Tabirian, A. V. Sukhov, B. Ya. Zeldovich, M.C.L.C. **136**, #1, pp. 1-140, 1986.
- I. I. Goosev et al., JETP Lett., **55**, p. 178, 1992.
- I. C. Khoo et al., Optics Lett., **20**, p. 130, 1995.
- N. V. Tabirian, A. V. Sukhov, B. Ya. Zeldovich, JOSA **B**, **18**, pp. 1203-06, 2001 (well-saturated regime at pump level about 0.5 Watt for  $\lambda = 1.06 \mu\text{m}$ )

# Coupled wave equations for the waves of two orthogonal polarizations

$A(z, t)$ , **pump amplitude**

$\theta(z, t)$ , **grating amplitude**

$B(z, t)$ , **signal amplitude**

$$\frac{\partial A(z, t)}{\partial z} = i \theta^* (z, t) B(z, t);$$

$$\frac{\partial B(z, t)}{\partial z} = i \theta(z, t) A(z, t);$$

$$\frac{\partial \theta(z, t)}{\partial t} + \Gamma \theta = A^* (z, t) B(z, t).$$

## Steady-state gain of the signal: energy transfer from the pump

Monochromatic Stokes shift  $\Omega$   
of the signal B with respect to pump A,

$$B(z, t) = \exp[-i(\omega_0 - \Omega)t]B(z)$$

$$A(z, t) = \exp[-i\omega_0 t]A(z)$$

$$\theta(z, t) = \frac{A^*(z, t)B(z, t)}{i\Omega + \Gamma}$$

yields

1) gain of the signal, and

2) cross-phase modulation of the signal:

$$\frac{dB(z)}{dz} = \frac{\Omega + i\Gamma}{\Omega^2 + \Gamma^2} \cdot |A(z)|^2 B(z)$$

# **This work: instability of the pair of monochromatic coupled waves**

**We are looking for the solution in the form:**

$$A=A_0\exp(ivz)\cdot[1+\alpha(z, t)],$$

$$B=B_0\exp(i\mu z)\cdot[1+\beta(z, t)],$$

$$\mu = |A|^2 / \Gamma, \quad \nu = |B|^2 / \Gamma,$$

$$\theta(z, t)=(A_0^*B_0/\Gamma)\cdot\exp[i(\mu-\nu)z)]\cdot[1+\psi(z, t)].$$

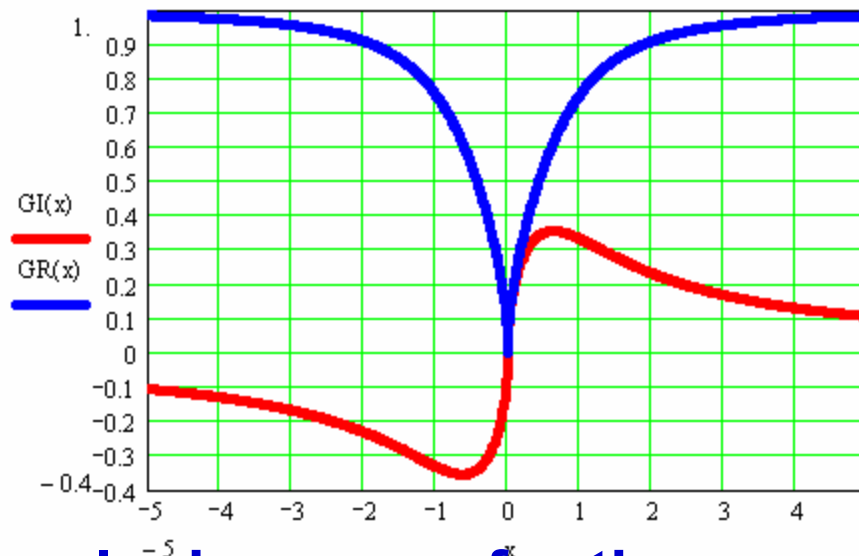
**Small perturbations  $\alpha, \beta, \psi$  are sought in the form**

$$\exp(i\Omega t+i\Lambda z).$$

# Instability of the pair of monochromatic coupled waves (contd.)

$GR(x) \equiv \text{Re}\{ \Lambda(\Omega=x\cdot\Gamma) \}$  is cross-phase modulation of the perturbation;

$GI(x) \equiv \text{Im}\{ \Lambda(\Omega=x\cdot\Gamma) \}$  is the spatial growth coefficient of the perturbation.



Example:

$$\mu = |A|^2/\Gamma = 0.65,$$

$$\nu = |B|^2/\Gamma = 0.35$$

**Mini-conclusion:** even for the monochromatic pair of waves, there is temporal 1-dimensional instability !

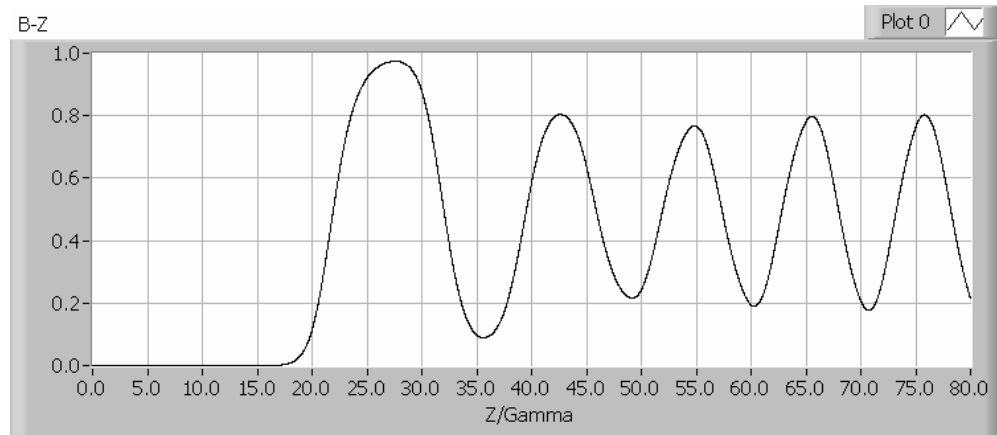
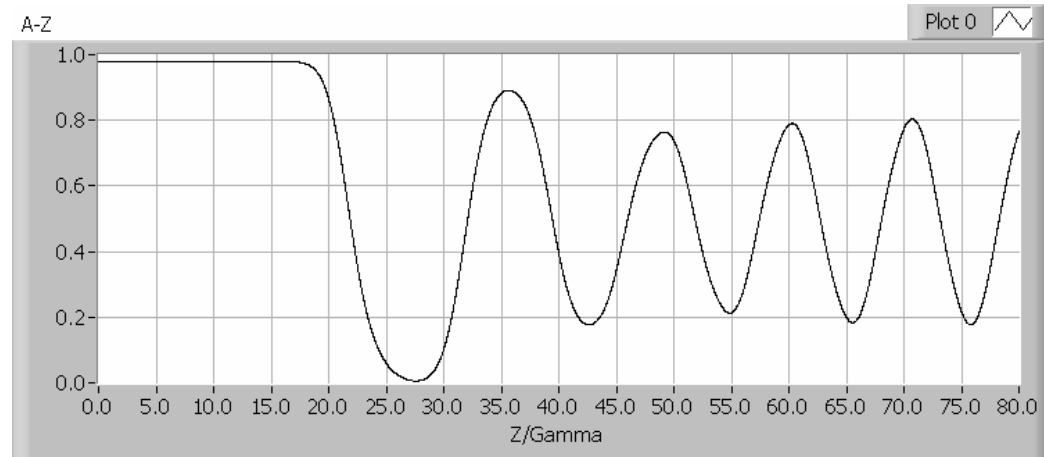
# Instability of the pair of monochromatic coupled waves: numerical modeling.

1. The case of no thermal scattering (no noise).

Example: Monochromatic waves,

$|A(z=0)|^2 = 0.99$ ,  $|B(z=0)|^2 = 0.01$ ,  $\Gamma t_{\max} = 25$ .

See next slide for instantaneous spatial distribution of intensities.



# Instability of the pair of monochromatic coupled waves: numerical modeling.

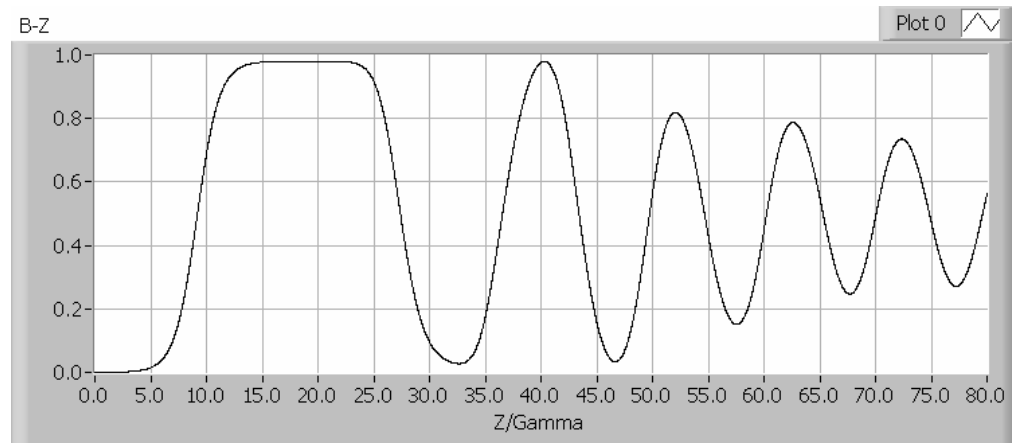
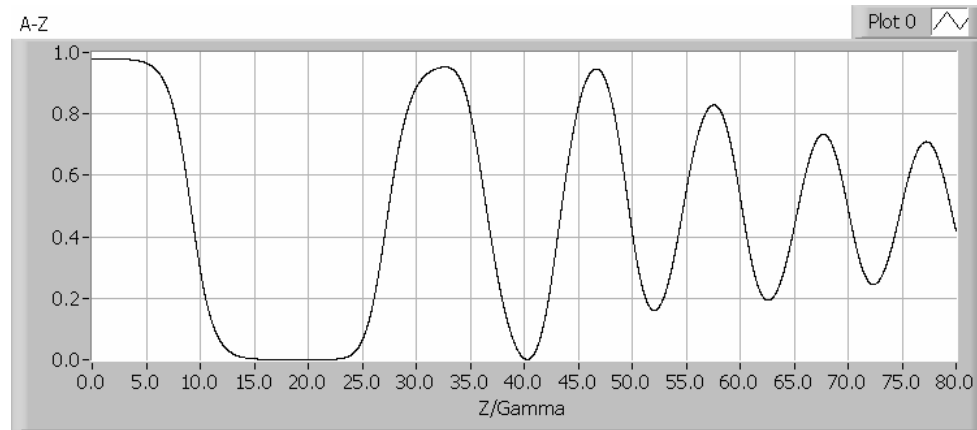
**2. The case of no thermal scattering (no noise).**

**Example: B-signal is frequency-shifted at the input,  $\Omega = x \cdot \Gamma$ ,  $x = 1$**

**$|A(z=0)|^2 = 0.99$ ,  $|B(z=0)|^2 = 0.01$ ,  $\Gamma t_{\max} = 25$ .**

**See next slide for instantaneous spatial distribution of intensities.**





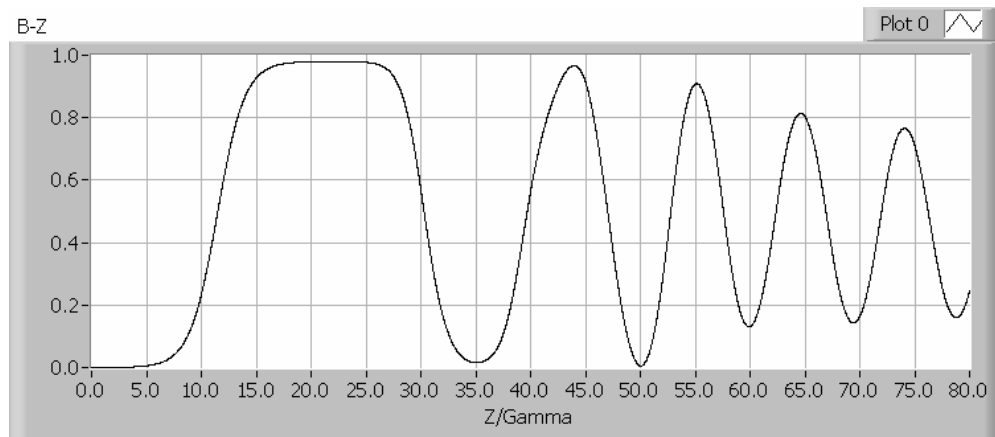
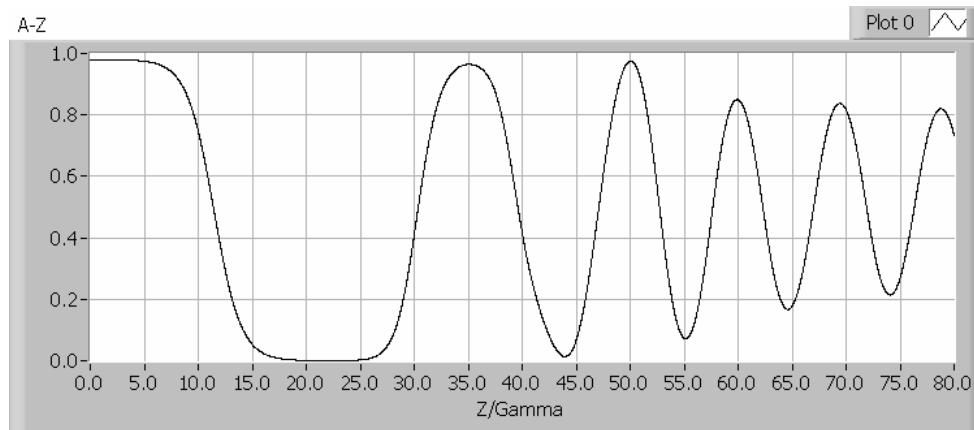
# Instability of the pair of monochromatic coupled waves: numerical modeling.

**3. The case of no thermal scattering (no noise).**

**Example: B-signal is frequency-shifted at the input,  $\Omega = x \cdot \Gamma$ ,  $x = 2$**

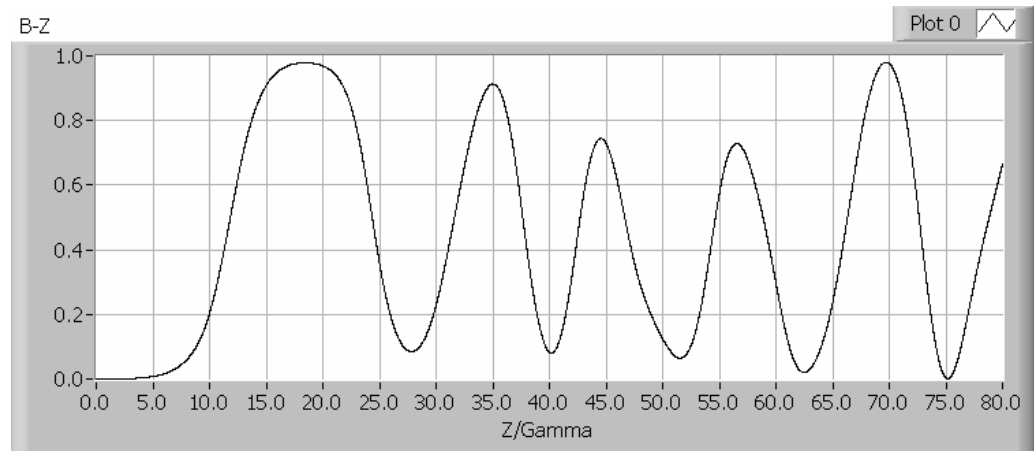
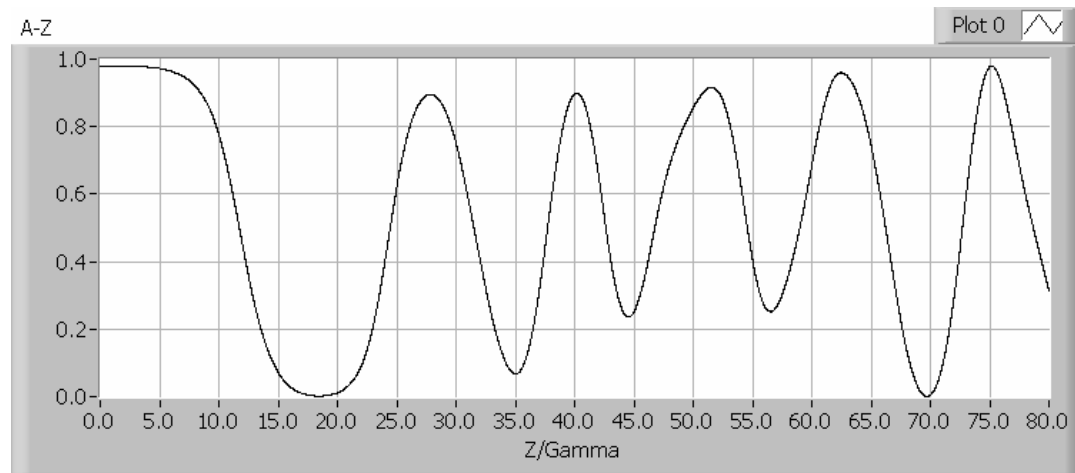
**$|A(z=0)|^2 = 0.99$ ,  $|B(z=0)|^2 = 0.01$ ,  $\Gamma t_{\max} = 25$ .**

**See next slide for instantaneous spatial distribution of intensities.**



## Instability of the pair of monochromatic coupled waves: numerical modeling.

4. The case of thermal scattering (with noise).  
Example: B-signal is frequency-shifted at the input,  $\Omega = x \cdot \Gamma$ ,  $x = 2$ , noise=0.05 (a. u.)  
 $|A(z=0)|^2 = 0.99$ ,  $|B(z=0)|^2 = 0.01$ ,  $\Gamma t_{\max} = 25$ .  
See next slide for instantaneous spatial distribution of intensities.



# Conclusion

1. Grating optical nonlinearity in Nematic Liquid Crystal is studied theoretically.
2. Instability analysis is performed analytically for the pair of monochromatic waves.
3. Numerical (z + t) model of two-wave interaction is developed for the analysis of energy transfer and cross-phase modulation.
4. Optimum choice of frequency shift  $\Omega = x \cdot \Gamma$  and of the input intensity of the signal allows for good beam clean-up !